Efficient Steering of a Catamaran Using Ackermann Principles

Ackermann steering principles were first applied to carriages in the 18th Century and afterwards to all manner of vehicles and trailers. This note looks at how the Ackermann geometry can be applied to catamarans. The Ackerman geometry solves the problem of how wheels (or in this case rudders) should be aligned when turning in a circle where the outside wheel (rudder) is turning on circle of a larger radius than the inside one. If the wheels are parallel to each other during a turn, one or other will skid over the surface causing unnecessary wear; if the rudders are parallel to each other, one or other will have turbulent flow over them causing loss of steering efficiency. More efficient turns are achieved if the inner wheel (rudder) is turned more than the outer one. Of course, when going in a straight line the wheels or rudders should be parallel to each other.

In catamarans, the Ackermann principle is applied by having the tiller arms toed-in by a number of degrees (typically between, say, 5^0 & 20^0). This is achieved by having the tiller bar shorter than the rudder spacings – see Figure 1. When the rudders are turned, the one on the inside of the turning circle will actually rotate through a greater angle than the one on the outside of the circle – by an increment which depends on the amount of toe-in.

This note describes the Ackermann geometry and addresses the issue of how much toe-in is required to approximate the Ackermann geometry.

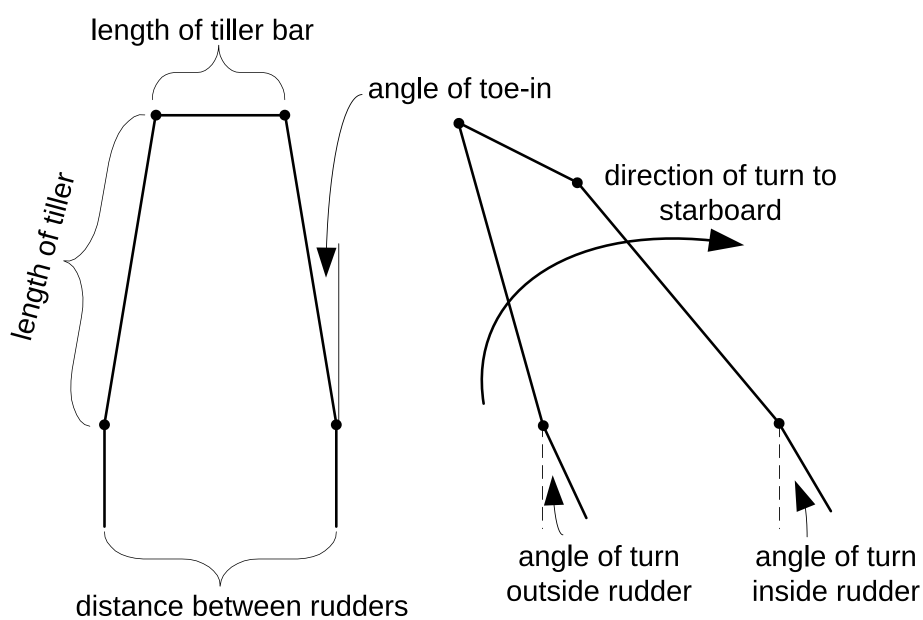

Figure 1: Ackermann Steering on Twin Rudders in a Catamaran

The Ackermann Geometry for a Catamaran

In Figure 2, a catamaran of length L and hull centreline spacing W is executing a starboard turn of radius R. Both rudders will be working at maximum efficiency if they lie on a tangent to a circle with the same centre as the turning circle. The flow of water over the rudders will then be laminar.

A line drawn from the centre of the turning circle to the pintle of the outside rudder makes an angle β with the direction the catamaran is moving in. A line drawn from the centre of the turning circle to the pintle of the inside rudder makes an angle α with the direction the catamaran is moving in. In the phrase beloved of mathematicians, “by inspection” (in other words if one looks at Figure 2 long enough and hard enough) one can see that the outside rudder is turned through 90-β and the inside rudder is turned through an angle 90-α; the inside rudder is clearly turned through a greater angle than the outside one. Also the angle made by the line of both rudders extended forward is β-α, which is the Ackermann angle, sometimes called the steering compensation angle.

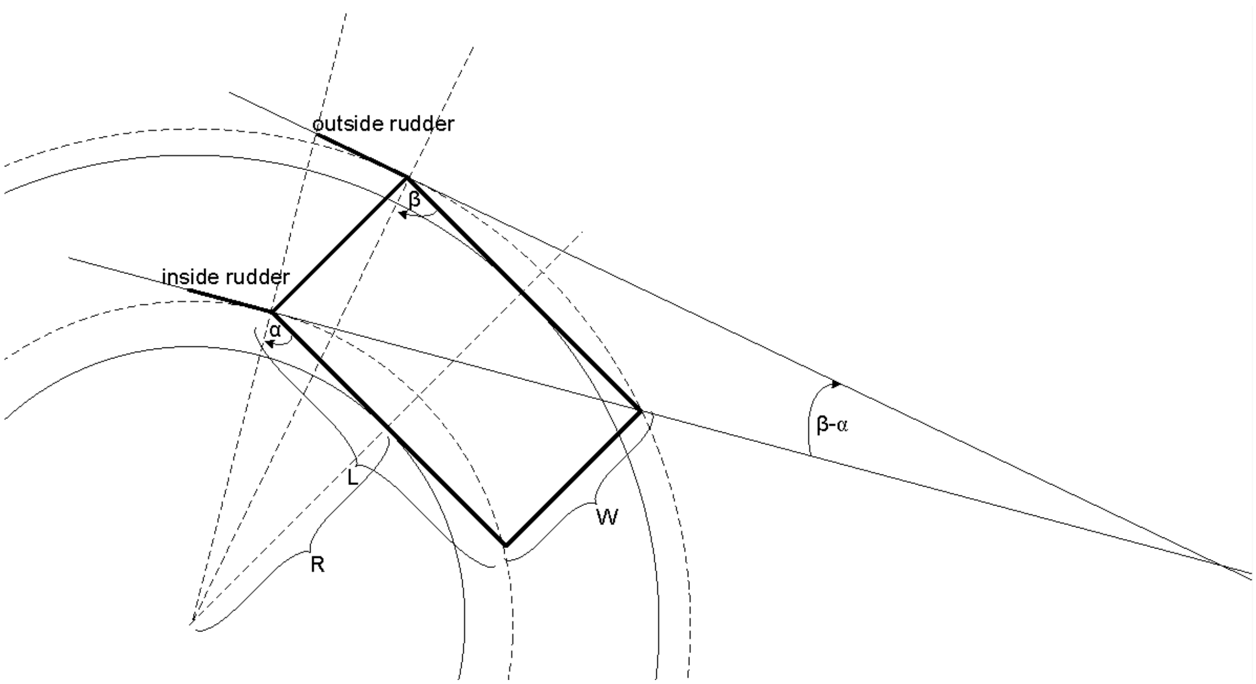

Figure 2: The Geometry of a Catamaran Turning in a Circle

Note that the Ackermann angle, i.e. the difference in the turning angles of each rudder which has both rudders working at maximum efficiency, will vary according to the radius of the turning circle.

The calculations are easy: $\alpha = \tan^{-1}(R/0.5L)$, $\beta = \tan^{-1}((R+W)/0.5L)$. The Ackermann angle for various turns, as the radius R is varied, are given in a spreadsheet shown in Table 1. The figures for the length of boat and distance between the rudders are taken from the Richard Woods design of the Eclipse catamaran.

distance between rudders			4400	mm	
length of boat			9900	mm	
radius of turn (mm)	α (degrees)	β (degrees)	Ackermann Angle (degrees)	angle inside rudder (degrees)	angle outside rudder (degrees)
4000	39	59	21	51	31
4500	42	61	19	48	29
5000	45	62	17	45	28
5500	48	63	15	42	27
6000	50	65	14	40	25
6500	53	66	13	37	24
7000	55	67	12	35	23
7500	57	67	11	33	23
8000	58	68	10	32	22
10000	64	71	7	26	19
20000	76	79	2	14	11

Table 1: Ackerman Angle Calculations for an Eclipse Catamaran

Catamaran Steering Approximation to Ackermann

Let the amount of toe-in for a catamaran be γ degrees. If the outside rudder of a catamaran turns through a known angle ζ, one can work out the angle of turn of the inside rudder by finding the angles ε and δ shown in Figure 3.

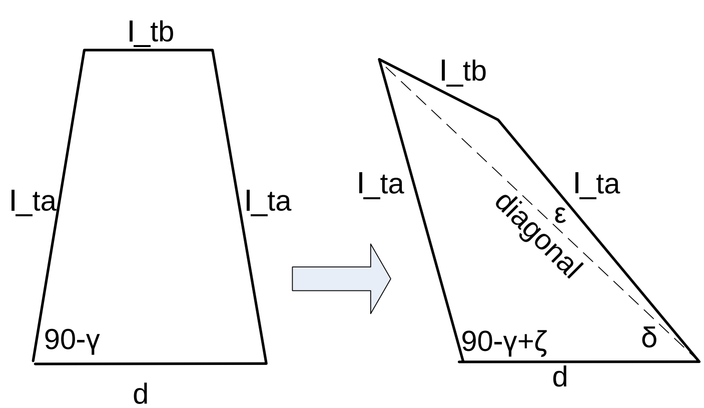

Figure 3: Angles Made by a Catamaran Steering System

To solve this requires application of a trigonometric equation in 3 stages. First find the length of the diagonal by solving the equation: diagonal = $(l\_ta^2 + d^2 - 2*l\_ta*d*\cos(90-\gamma+\zeta))^{1/2}$. Now one knows the length of all sides of both triangles formed by the diagonal, use the same equation twice more to find the angles ε and δ. The angle of turn of the inside rudder is 90-γ-δ-ε.

It is instructive to put these equations into a spreadsheet and play around with the angle of toe-in. In Table 2, I have a toe-in of 5^0 and I have input into the spreadsheet the same angles for the rudder on the outside of the turn as we have in Table 1. The spreadsheet calculates the angle of rotation of the inside rudder, given this steering linkage design. The difference between the angles of rotation of the inside rudder in Table 2 and Table 1 show how good the approximation is to Ackermann – the lower the number the better. Note that with this toe-in we have a very poor approximation to the Ackermann ideal.

	distance between rudders, d			4400	mm
	length of tiller arm, l_ta			1100	mm
	angle of toe-in			5	degrees
	length of tiller bar, l_tb			4208	mm
diagonal	δ	ε	angle inside rudder (degrees)	angle outside rudder (angles)	differs from Akermann by (degrees):
4974	12	41	32	31	19
4952	12	43	31	29	17
4931	12	44	29	28	15
4912	12	45	28	27	14
4894	12	46	27	25	13
4877	12	47	26	24	12
4862	12	48	24	23	11
4847	12	49	24	23	10
4833	13	50	23	22	9
4786	13	53	20	19	7
4654	14	60	12	11	2

Table 2: Ackermann Approximation when Toe-in = 5^0

If we increase the toe-in to 20^0, see Table 3, we have a much better approximation.

diagonal	δ	ε	angle inside rudder (degrees)	angle outside rudder (angles)	differs from Akermann by (degrees):
4726	13	10	47	31	4
4701	13	15	42	29	6
4677	13	18	38	28	6
4656	14	21	36	27	6
4636	14	23	33	25	6
4617	14	25	31	24	6
4600	14	26	30	23	6
4583	14	28	28	23	5
4568	14	29	27	22	5
4516	14	33	23	19	4
4374	14	43	13	11	1

Table 3: Ackermann Approximation when Toe-in = 20^0

Discussion

To get a good approximation to Ackermann, the degree of toe-in required on a catamaran tiller steering system has to be large – 15 to 20 degrees. A good approximation to Ackermann will make the catamaran's steering feel responsive; with only 5^0 of toe in, I would be surprised if it were possible to notice any difference compared with having no toe-in at all. This link http://hem.bredband.net/b262106/Boat/acker.html provides another analysis of catamaran steering with much more complicated mathematics than I have used, but comes to the same conclusion.

This article by Tom Speer (http://hem.bredband.net/b262106/Boat/TSpeer_acker.pdf) is interesting but I find it a little baffling. He approaches the catamaran steering problem from the point of view of fluid dynamics, deriving equations from such variables as lift, drag angular momentum and the two rudders' angles of attack. I have studied it for a while but it is a bit beyond me I'm afraid!

I do have a couple of observations though. Firstly he equates the Ackermann angle with toe-in which, in my opinion, it isn't. Secondly, it is worth quoting his conclusion:

> *"But it appears that for the geometry assumptions made so far, for maximum leeway and rudder angles of attack on the order of 8 - 10 degrees the minimum turning radius would be on the order of one to two beam-widths and the most suitable Ackermann angle* (i.e. toe-in) *would be fairly small - on the order of 10 degrees to 15 degrees, depending on the maximum angle of attack. Too much Ackermann and the starboard rudder reverses at the high rudder deflections needed for tight turns."*

I think this conclusion is consistent with the discussion here. My geometrical analysis would suggest that angles of 10, 15 or 20 deg are more efficient than 5 deg. Tom Speer, approaching the problem from a different perspective, is saying the same thing.

Of course, the greater the toe-in angle, the shorter the tiller bar. If the tiller bar is too short you might not be able to execute a tight turn at all. For example, in Table 3, an inside rudder angle of 47^0 is close to the maximum possible – in a marina berthing situation you might want to angle the rudders even more than that and to hell with Ackermann.

With large toe-in angles you run out of linkage when turns are tight – as Speer says, the inner rudder "reverses". Speer says this happens when the turning circle is about 1 – 2 beam's widths (I think he defines the beam as being the distance between the hull centrelines). This agrees with my analysis – with a toe-in of 20 deg, the Eclipse runs out of linkage when the turning circle is one of Speer's "beams". I think it prudent, therefore, to go for a toe-in of less than 20 deg, but more than 5 ; 10 to 15 deg would strike me as about right.

For a discussion of these issues on a forum see:
http://www.freeforum101.com/woodsdesigns/viewtopic.php?t=1670&postdays=0&postorder=asc&start=25&mforum=woodsdesigns .